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These materials were written with the aim of reflecting the thinking of The Cambridge Conference on School Mathematics (CCSM) regarding the goals and objectives for school mathematics. Presented are plans for teaching 15 inequality lessons for above average first grade students. The discovery approach is utilized by the teacher in order to involve students in the classroom discussions. Ten wooden sticks of varying lengths and thicknesses were available for these inequality lessons and they were used by both teachers and students in experimental situations. Activities and comments by the teacher and the students are provided for each lesson. [Not available in hard copy due to marginal legibility of original document]. (RP)

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"Inequality" Lessons at Adams School, Lexington

February 28, 1967 Adams School Above Average - Grade I

Class Teacher: Judy Morecz, Organized and taught by Mrs. B. Fitzgerald, Co-ordinator of K-3 Mathematics, Lexington Schools

Session I Teacher: B. Fitzgerald - Comparison of sticks.

Materials: 10 wooden sticks of varying lengths and thicknesses, strips of colored paper were fastened to each stick.

Color	Symbol	Length
dark green	d	25"
black	k	20"
yellow	y	18"
red	r	17"
orange	o	14"
purple	p	12"
blue	u	10"
green	g	9"
white	w	8"
brown	n	5½"

The same color names and their symbols names were used as when the children used Cuisenaire rods.

Since the children had used the symbols < and >, this presented no problem when being adapted to this situation.

The sticks were in a wooden container at the front of the class.

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T. Today we are going to talk about these sticks. I'm going to choose two sticks. Tell me what you see.

S. a. One is bigger than the other.
b. They are different colors
c. The yellow stick is curly. The other stick isn't.
d. Both the sticks are straight.
e. There's a lump at the top of the purple one.
f. The yellow one has more bumps.
g. One is sticking out longer.
h. When you are holding them up, they are not the same.
i. One is bigger and one is smaller.
j. The yellow is bigger than the purple.

T. Can some one write that on the board for us using this symbol $>$?

S. Writes on the board: $y > p$

T. Now I need two helpers to choose two more sticks.

S. James and Leslie get sticks and hold them up for class to view. Class discusses sticks as above. Ruth writes on board: $g > r$

T. Let's have two more helpers choose some other sticks.

S. Sandy and Abby select k and o sticks.
a. The smaller one is thinner than the big one.
b. The black one is bigger than the other one.

T. Who can write this on the board for us?

S. a. Pupil writes $k > o$

T. Who can write this another way?

S. a. Pupil writes $o < k$
Pupils read notations on the board. ($k > o$, $o < k$) Compare sticks to verify.

T. We need two other helpers.

S. Linda and Eric select u and g sticks.
a. They are the same size. (Children voice variance of opinions) Two helpers compare sticks. Pupil a. now writes on the board $u > g$. Class reads this together from board)

T. Two other helpers may choose sticks now. (Ann and John get two remaining sticks.) Let's use these symbols h for brown and ω for white.

S. a. One has no color. (Stick was all white.)

T. This is a white stick without any strips of paper. So it really does have color.

S. a. Brown is smaller.

T. Raise your hand if you agree.

S. a. Let's measure them. (Children compare n and ω) b. I can write it on the board. n < ω

T. Can some one use this other symbol. Points to > on board.

S. a. Child writes ω > n and reads this to class.

T. Let's read everything we have written on the board about our sticks. (Reviews all notation) Which children have not had a turn to use the sticks? Noreen and Jon may get a stick. What can you tell me about them?

S. a. The g one is bigger than the u one.

T. Here is another stick (K)

S. a. This one is bigger than the other two.

T. Let's have two children who haven't had a turn get sticks now.

S. a. The purple is bigger than green.
b. The green is smaller than the purple.

T. What can you tell me now with this third white stick?

S. a. White is the smallest.
b. Put them together and see.

T. These two children will be the last to use the sticks today. (Children select d and r).

S. a. The green is longer than the red.

T. Who can show me how to write this?

S. a. Writes on board d > r

T. Look at my g stick.

S. a. The dark green is bigger than g and r.

b. Measure them together.

T. Who can write this on the board?

S. a. $d > g > r$

T. Let's take a look at the sticks and see. Read this at the board $d > g > r$.

LEXINGTON PUBLIC SCHOOLS Lexington, Massachusetts

March 10, 1967 Adams School Above Average - Grade I

Classroom Teacher: Judy Morecz

Session II - Teacher: B. Fitzgerald

Objectives: 1. Establish Two Stick Rule with class
2. Worksheet on Two Stick Rule to check individual mastery
3. Begin game to develop Three Stick Rule

Materials: Same wooden sticks as described in Session I and two large shopping bags.

Note: This lesson was originally scheduled for Tuesday, March 7, but had to be postponed due to a snowstorm and no school.

Prior to my coming, the regular classroom teacher, Judy Morecz, had spent about ten minutes doing some similar activities as in Session I.

T. Tell me what you can about these sticks. Holds p and n)

S. a. P is bigger than n. (Records on chalkboard) $p > n$.

T. Who can use this symbol with the same sticks?

S. b. Records on chalkboard $n < p$. Then reads aloud to the class $n < p$.

T. Let's all read what the children have written and see if you agree by looking at the p and n sticks: $p > n$ and $n < p$. (Class agrees)

T. Now write 2 things that are true about these sticks (y and o) and arrange it like this for me using these symbols correctly:

$$> : <$$

S. c: Writes on board

$$y > o : o < y$$

T. Now let's all read this together and see if it is correct. (All agree)

T. Can you fill in the symbols using these sticks (u and r)?

S. a: The $r > u$ and the $u < r$. (responds orally)

T. Can you fill in the symbols on the board?

$$> : <$$

S. a: Records on board

$$r > u : u < r$$

T. Can Lesley develop for us the two stick rule? (Points to above notation just written on board.) This is what we call the 2 stick rule. Lesley, can you tell us what we mean by the 2 stick rule?

S. a: You use 2 sticks.

T. What are we doing?

S. b: Measuring.

T. What do we find out?

S. c: We find out how big they are.
d: If the ↾ > ∪ the ∪ < ↾

T. Let's all read the two stick rule we just wrote.

Worksheet #1 is distributed to the class after simple introduction used at board.

T. This paper is all about the two stick rule. Write your first and last name on the paper. Everybody look where it says 1 on the paper. We'll work together on the first row. Put your finger on the "1". We'll work here with these 2 symbols ơ and ɣ (Records symbols on board. Then holds up ɣ and ơ sticks.) Let's see if everyone can write the two stick rule at the left and then at the right. Look up when you finish. (Teachers make quick check of class.) Read the two stick rule you wrote on your paper.

S. All read together ɣ > ơ, ơ < ɣ

T. Let's go down to #2. (Holds up n and ω.)

Note: Children quickly filled in symbols. Then there was an audible note of dismay as some <u>just</u> discovered the symbols had been turned around. Erasers were used and sought by others. Others had recognized this change before recording on their papers.

T. Look at the signs again in Row 2. They are not the same as in Row 1. I tried to trick you! (The rest of the worksheet was completed quickly with no other comment on directions of symbols.)

T. Is everyone ready to do the next thing with me? We're going to play a game with these two big bags. I need some helpers. For now, I'm going to put all the sticks into one bag. I'll explain this stick game to you. This one will be called "The Comparison Stick" (light green - g). I'll take a helper outside our classroom with me. The helper will come in and tell us how the stick in one bag compares with this comparison stick. Do you know what I mean by comparison.

S. a: You compare the sticks and measure them together.

T. (Goes outside classroom door with all materials and places p stick in one bag, the w stick in the other bag, and covers all other sticks with large paper. Meanwhile, other T. in classroom has chosen two helpers and Jeff to hold Comparison Stick.)

T. Julie come outside with me.

S. a: Julie returns and writes on board after looking at Comparison Stick: $w < g$

b: All pupils read aloud Julie's recording.

T. We're trying to find out what we can about the sticks in the bag. Julie, can you write it the other way using the two stick rule?

S. Julie records $g > w$.

T. Julie is writing the two stick rule for us now.

S. All read together $w < g \; ; \; g > w$

T. Sandy will pick a stick now. (T. and Sandy go out to bag 2.) Sandy returns, looks at Jeff holding Comparison Stick at front of room. Then writes $g < p$

T.. Write it over here on the board using the two stick rule.

S. a: Sandy writes: $p > g$

T. What color was the stick in the other bag?

S. All respond - p

T. Let's read Sandy's rule together.

S. $g < p, \quad p > g$

T. How do w and g compare?

S. $w < g$

T. How do p and g compare?

S. $P > g$

T. Think of the 2 sticks in the bag outside. How do the two sticks in the bag compare with each other?

S. a: P is bigger than g.

T. Think again. We know (points to board) $P > g$ and $\omega < g$. If I use the two stick rule, what else could I say about P and ω?

S. a: $\omega < P$

T. How do you know that $\omega < P$?

S. a: $\omega < g$, $P > g$, so $\omega < P$.

T. I should like to introduce to you a new symbol to write this

$g < P,\ \omega < g \Rightarrow \omega < P \Rightarrow$

This says: it follows that $\omega < P$. I'll read it again and then you can read it with me.

S. All read in unison

$g < P,\ \omega < g \Rightarrow \omega < P$

T. Maybe it would be a good idea to get the bags and find out if this is true. Jeff, will you get the sticks and prove it to us?

S. a: (Jeff returns to classroom with ω and P stick.)

T. Let's say together what's on the board and while we read, I'll show you. (S. read - T. holds up each stick to prove equation as its read.)

T. What was this stick (g) called?

S. Comparison Stick.

T. What does $\Rightarrow$ mean before I leave?

S. It follows.

T. This is called the three stick rule. We'll play it again next time.

LEXINGTON PUBLIC SCHOOLS LEXINGTON, MASSACHUSETTS

Tuesday, March 14, 1967 Adams School Above Average - Grade 1

Classroom Teacher: Judy Morecz

Session III - Teacher: B. Fitzgerald

Objectives: 1. Work on Three Stick Rule

2. Observe if Three Stick Rule will always work

Materials: Two large shopping bags and some wooden sticks but colored papers have been changed to:

w = 4"		u = 14"	
n = 5½"		r = 17"	
y = 9"		g = 18"	
o = 10"		k = 20"	
p = 12"		d = 25"	

T. I want you to think about the last time I was here. What were we talking about?

S. a. Sticks that are small and big.

b. The Three Stick and Two Stick Rule.

c. We were playing a game.

T. Today we'll play another game called "Hiding in the Bag". I'm going outside to hide some sticks and Miss Morecz will choose 3 helpers. (Linda is chosen to hold Comparison Stick-u in front of classroom.)
Debbie looks at Comparison Stick (u), then goes outside to one of the bags. Returns and writes $r > u$.
Peter does same - returns and writes $y < u$

T. I'd like to have everyone read what Debbie and Peter have written.

S. (All read 2 phases written on board.)

T. Which two sticks are hiding in the bag?

S. Red and yellow.

T. Look at what Linda is holding and look at what is written on the board. What can you tell me about the two sticks that are outside?

S. a. $r > y$

T. How do you know?

S. If r is bigger than v and y is smaller than blue, then red is bigger than yellow.

T. Very good. Now we'll write the new symbol.
Who remembers what it is?

S. "It follows" and it's like this $\Longrightarrow$

T. (Writes on board) $r > v,\ v > y \Longrightarrow r > y$.
Who can read this to us?

S. a. (Reads blackboard notation to class.)

T. Let's have my helpers get the 2 sticks in the bag that are outside our door. We'll see if we're right.

S. (Two helpers exit and return with r and y sticks.)
Debbie, Peter, and Linda stand sticks up in chalk tray as class compares sticks and rule written on board.

T. Let's all read the whole rule together. Do you think this rule will always work?

S. a. Yes
b. Maybe.

T. I'll put 2 more sticks in the bags while Miss Morecz chooses three new helpers.

S. a. (Holds o for Comparison Stick.)
b. Exits to get stick.

T. Let's see if our rule will work again.

S. a. Returns from outside and writes on board $n < o$.
b. Next child returns and writes $k > o$.

T. Who knows the names of the 2 sticks outside?

S. a. Black and brown
b. All read from board $k > o,\ n < o$

T. What can you tell me about the 2 sticks in the bag?

S. a. $K > n$

T. Who can write the whole of the Three Stick Rule for us?

S. a. Writes on board: $K > o,\ o > n \Longrightarrow K > n.$

T. Let's have our helpers get the sticks and see if he's right.

S. Return with sticks to front of blackboard and holds all 3 sticks to compare and check.

T. Did the Three Stick Rule work this time?
Will it always work?

S. a. Yes

b. Some say no

T. Let's try once more now. Jean will hold the Comparison Stick (r).

S. Jan looks at Comparison Stick, goes outside to bag then comes in and writes $d > r$.

T. Let's all think what might be in the other bag while Ruth goes out to look.

S. a. Ruth returns and writes $g > r$.

b. Class reads $d > r$ and $g > r$.

T. What 2 sticks are outside?

S. g and d

T. What can you tell me about the 2 sticks outside?

S. a. $d > g$

b. $d > r$

c. $r < g$

T. What can we write now?
(Children wait)
Kathy get the 2 sticks in the bag for us.

S. a. Holds up d and g.

b. Holds r stick.

T. Can we say $g < d$?

S. Yes.

T. We said $d > g$ and $g < d$. Can both be true? Can we write out a Three Stick Rule this time?

S. Jan gives his rule as:
$d > g,\ g > r \implies r < d$

B. All read Jan's rule.

T. See what happened to our Comparison Stick. Did our Three Stick Rule work?

S. The Comparison Stick was the smallest.

T. Does it always work?

S. a. Yes

b. No

T. Do you know why it does not always work? Don't tell if you know. I'll leave you with this question.

Comments: In the last game we played, it was suggested $g > d$. I should have tried to develop this further. Student (Jan) did come up with a good equation regardless of the variation in our game. Student's comment "Comparison Stick was the smallest" was correct and needed more explanation. We need more variations to rule.

LEXINGTON SCHOOLS LEXINGTON, MASSACHUSETTS

Tuesday, March 21, 1967 Adams School Above Average - Grade 1

Session IV Teacher: B. Fitzgerald

Teacher: Judy Morecz

Objectives:
1. Work on Three Stick Rule with class.
2. Worksheet on Three Stick Rule to check individual understanding.
3. Use game with class. Can they explain why Three Stick Rule won't work.

T. Today we're going to play our game "Hiding in the Bag" and use our Three Stick Rule. Each of you will have a worksheet to put your answers on. I shall help you with the first row called A. (Papers are distributed with "mechanics" explained. Teacher F1 and Teacher M2 display sticks from opposite sides of the room while Comparison Stick is displayed in chalk tray at the front of the room.)

T. What is this stick called?

S. Comparison Stick (o).

T. This is my stick - n. What shall we write on our papers?

S. $o > n$

T. Now T2 will hold up another stick - r . What shall we write on the next part of our paper?

S. $r > o$.

T. What can you tell me about the sticks in each bag?

S. It follows that $r > n$

T. Now I'll write the first row on the board so we can all check our papers.

$o > n,\ r > o, \Longrightarrow r > n.$

Look at the board. Then look at your paper. Let's all read this rule from the blackboard.

T. Everyone close his eyes while we change the sticks.
Now this is the Comparison Stick (r). Go to row 2. I will not help you. Write just what you see and think. Look at my stick (g). Take a good look. Write it on your paper in Row 2. I'll put it nearer to the Comparison Stick.
T2 holds up q stick.
Write what you think. Place nearer r . Now $\Longrightarrow$ ... you finish it by yourself. Think of the two sticks in the bag.
Pencils down. Children close their eyes while teachers rearrange sticks.
Rows 3-5 same procedure as previously.

3. $y > w,\ p > y, \Longrightarrow p > w$
4. $d > K,\ K > r, \Longrightarrow d > r$
5. $n < p,\ p < g \Longrightarrow n < g$

Worksheets collected. All stand.

T. How many think the Three Stick Rule will always work?
(One student, some skeptical and others undecided.)
Let's play our game "Hiding in the Bag" once more today. (Bags with sticks are outside the classroom)
Cynthia may hold the Comparison Stick - d. Look at what she's holding.
John, come out and look in a bag but first look at the Comparison Stick.

S. John returns and writes $d > y$.

T. What stick is outside?

S. a. Yellow
b. Class reads $d > y$

T. Now Ina may come outside.

S. Returns and writes $d > r$

T. What 2 sticks are outside?

S. y and r

T. Now think .... what can you tell me about y and r, the two sticks which are outside?

S. $r > y$

T. Why? How do you know?

S. It's just what I think.

T. What do you think (another child)

S. $y > r$

T. How do you know?

S. I don't know. I'm just guessing.

T. What do you think?

S. Last time I saw the y it was bigger than r.

T. What do you think?

S. $d > r$

T. There's something else I'd like to know.

S. $d > y$

T. I'm asking about the sticks in the bag outside our classroom.

S. $r > y$

T. How do you know?

S. a. I saw the sticks.
b. $r < r$

T. Yes, but what about the sticks outside?

S. $y < r$

T. Why? Does it tell that here on the chalkboard?

S. a. No
b. $r > y$. I just think it.

T. Look here at the chalkboard - Does it tell you which is bigger?

S. No. I think they are the same.

T. What do you think?

S. I changed my mind and think $r < y$.

T. Why?

S. ?

T. What do you think?

S. The last time I saw it r was bigger.

T. I'm going to stop here and not give my secret away.

Lexington Public Schools Lexington, Massachusetts

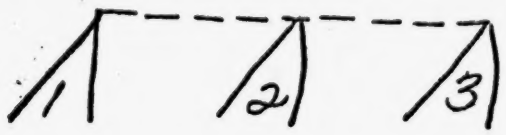
Tuesday, March 28, 1967 Adams School Above Average - Grade 1

Classroom Teacher: Judy Morecz

Session V - Teacher: Dr. Lomon

Materials used: 10 wooden sticks, and tape recorder

T. We have been working on the sizes of things to find out which is bigger and which is smaller. I have a problem and I don't know the best way to handle it. What you have been learning should be able to help me, I think. I'll ask you some questions to show you what my problem is to see if we can can get some answers. My problem is very strange. I have a mapping problem about mountains. We have to find out which are the higher mountains. Planes have to go over them. People have to build bridges here and there. The country is so rough that people cannot bring up very good measuring instruments. I send my men up there to look at the next mountain. They climb to the top of the mountain and look at the next mountain. They look up or down to see if what they see is higher or lower than they are. They cannot see all the mountains from one mountain as another mountain gets in the way - Here's my problem - (Draw on board)



Here's my man over here (1). He's looking at the next mountain. What do you think he says as he looks at the next mountain over there?

S. It's bigger.

T. He says it's bigger because he has to look up. Another man has climbed very high and gotten to this mountain (2) First he looks over to this mountain.

S. It's smaller.

T. He says that mountain is smaller. The men are going to have the mountains named after them. (Label mountains)

Bill Tom Bob's

Julie said that Tom's mountain was bigger. Can you write this for me?

S. Bill $<$ Tom.

T. Does everyone think that rule applies correctly?

S. Yes.

T. Let's write it a shorter way instead of using the whole name use the first letter of the name. B for Bill and T for Tom.

S. B $<$ T.

T. Now we have 2 B's. Let's call Bob's Rob. Use R instead of B for Rob.
When Tom got to the top of his mountain he said that Bill's mountain was smaller. How do we write that? What did Tom say about Bill's mountain.

S. a. B $<$ T b. T $>$ B c. B $>$ T

T. Everyone agree?

S. Yes...No

T. Why do you say no?

S. Because I can see it on the board.

T. The men cannot see 1 mountain from 3 mountain. They only carry a level. It is a little instrument with a water bubble in it. The water bubble tells them they are looking up straight and see the top of the mountain. The mountains are close and that's why they climb to the top of the mountain.

S. I skied at Mt. Washington with my brother. We climbed almost to the top. My mother's friend got sick so she didn't.

T. Look what we found out. R $>$ T, T $<$ R. Does that tell us one or two things?

S. One thing.

T. Why do you say that? Do you have a rule that tells you that?

S. If R $>$ T, it would be just the opposite that T $<$ R.

T. Do you have a rule for that.

S. a. We're talking about two mountains T and R's. It's something like the 2 stick rule.

b. I think it's more like a 2 mountain rule.

T. That's right. It's a two mountain rule. What about this between B and T?
T $\nearrow$ B, B $\angle$ T. Is that another case of the two mountain rule?

S. T $\nearrow$ B, B $\angle$ T.

T. It looks that way. Now here's my problem. Even if it's a nice clear day and Bill wanted to see Rob's mountain to tell if Rob's mountain were bigger or smaller than his, he couldn't see Rob's mountain.

S. Tom's mountain is in the way.

T. Yes, the peak of Tom's mountain top is in the way. That's pretty awful! What can we do about this? The airplanes want to know where they are going. Is Bill's mountain taller than Rob's or is Bill's mountain shorter than Rob's? Do we know? How can we find out? Kathy has an idea. Let's see how many have an idea. Keep your idea and whisper it to me. (Children whisper ideas individually to Dr. Lomon. These ideas all concerned getting around the restrictions.)

S. a. We can get an airplane.

T. It costs too much money to do this job.
It's only a small mountain. I have much less money.

S. b. Climb up Tom's mountain. It would tell us.

S. c. Can I tell if B's or R's is bigger?

S. d. Look both ways and use a yardstick to see which is bigger.

T. A yardstick atop a big mountain!

S. (giggle, giggle) Yes!

T. How many children think I could measure mountains with a yard stick?

S. a. It's too hard.

S. b. Maybe B could go down a mountain.

T. What would he find out?

S. R > T.

T. We have that here R > T. Tom says R's mountain is higher than mine. Bill already knows that. What does Tom know?

S. R > T.

T. Does Tom know anything else?

S. T > B.

T. I'll circle the two things Tom knows (T > B.) (R > T.) If he knows those two things does he know anything about Rob's and Bill's. Is R's > B's B's > R's ?
Look at the mountains. Can they see the tops?

S. a. R > T He saw it.

S. b. T > B He saw Bill's

S. c. R > B

T. Can Tom call down by telephone to tell me R > B? Should I believe him?

S. Yes, Yes.

T. Julie, do you believe him now?

S. a. Yes R > T and it must be bigger than Bill's because Tom's is bigger than Bill's and Rob's is bigger.

T. Does everyone agree?

S. a. Yes.

S. b. Can't Bill run down his and go up Rob's?

T. What would Bill find out that was new?

S. He'd see what was on top.

T. He'd find it good exercise. What kind of a mountain rule is that? We've had a two mountain rule and now we've learned something about Rob's mountain and Bill's mountain by using Tom's mountain. Could we have found out about Rob's and Bill's mountain without standing on Bill's mountain?

S. Rob could have climbed down and climbed up Bill's.

T. What would he be looking at?

S. He'd be looking at Tom's.

T. Yes. He'd have to look at it from here and from here. Or, the other way to solve my problem is we could walk to the top of Tom's and look at Bill's and Rob's.

S. Look at Bill's and Rob's.

T. Shall we give it a name?

S. The three mountain rule.

T. Does it work like the three stick rule?

S. Yes.

T. I have another mountain over here. How many mountains do we have now?
(Board diagram)

S. Four.

T. Rob can see it and it can see Rob but it can't see Bill and Tom. Only Rob can see it. I'll send Sandy up this mountain. (Labels it S)

S. a. (Sandy was temporarily "in the clouds" and immediately rejoined us.)

b. Giggle-giggle.

T. Sandy, you're up top of your mountain and you look at Rob's mountain and what do you see?

S. a. Rob's $\angle$ Sandy's. (Goes to board and writes.) $S > R$

T. What do we know now? (Review 2 stick rule from board)

$R > T$, $T > B$, $S > R$

We also know some other things that come from the 2 stick rule.

S. R > T

T. If R $>$ T what do we know about T's and R's?

S. T $<$ R

T. T $<$ R. The 2 stick rule is easy. What do we know about B's and T's.

S. B $<$ T

T. B $<$ T. It's the three stick rule - R $>$ T. Is Sandy's bigger or smaller than T's or B's? What happens now? Who can solve this problem?

S. Sandy could yell to Rob and ask him if his is $>$ T's and also ask him to ask Tom is his $>$ than Bill's.

T. Right. He could find out. Sandy yells mine is bigger than R's and R said mine is bigger than Tom's. So Sandy knows that. Tom looks at B and Tom said B $<$ mine.
T $>$ B Rob yells back to Sandy T $>$ B and Sandy looks and says S $<$ R. What does he know? What could he find out about S and T's.

S. S $>$ T

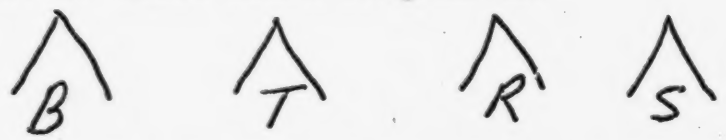
T. Does anyone think maybe S $<$ T?

S. No

T. Why do you say that?

S. R $>$ S, T $>$ R, so it has to be.

T. Let's check it with the sticks. (Sticks are placed in chalk tray in front of chalk board diagram as)



S. We can draw a line across from Sandy's to Tom's or Bill's.

T. Can we do that? He can't do it. He has to use what he knows about big and small sticks. We found out R $>$ B and S $<$ T. What about S's and B's.

S. a. S $>$ B

T. How do you know that?

S. a. I can't see the top of Bill's.

b. S > R and R > T so it would have to be that it is bigger than Bill's.

T. Let's check with 4 sticks. (See previous diagram just shown. Check sizes of 4 sticks with class to fit diagram.)

S. a. (Sandy) My mountain > than Bill's.

T. Which stick is bigger?

S. a. The K one K > O

T. The K stick is where S's mountain is.

The O stick is where B's mountain is.

Yes. S > B.

How many mountains to get from S's to B's?

S. Four mountains

T. Four mountains to compare? We have a four mountain rule! How many mountain rules can we get?

S. (in chorus) a. A hundred! b. A thousand! c. A zillion! d. A hundred thousand! e. As many mountains as we could count up to.

T. An awful lot! Maybe we can get a chance to find out some more things another time with Mrs. Fitzgerald.

Lexington Public Schools Lexington, Massachusetts

Tuesday, April 4, 1967 Adams School Above Average - Grade 1

Classroom Teacher: Judy Morecz

Session VI Teacher: B. Fitzgerald

Objectives: 1. Develop Five Stick Rule
2. Introduce Strategy Game that we named "Find My Stick"

Materials: Ten sticks, large bag and egg carton with holes to use as receptacle to hold sticks.

T. Who can remember what Dr. Lomon drew on the board?

S. Mountains.

T. What can you tell about them?

S. a. Some mountains were big.
b. Some mountains were small.

T. Today we'll have some mountains for boys and some for girls. We'll use names of boys and girls in this class. Watch! (Diagram on board.)

Let's read the names of the mountains together.

S. (Class reads in unison names on diagram.)

T. I'll call these children to come up and tell us about their mountains. When you write about your mountain, use only the first initial. We'll let the ladies begin. Ruth, you're on top of your mountain and you can only see the one next to you. Write what you see.

S. a. Ruth writes: $R > E, E < R$.

T. What rule are we using?

S. a. The Two Stick Rule

b. Class reads: $R > E, E < R$

T. Will Julie come up next and write what she can tell.

S. a. Julie writes $J > E,\ E < J$

T. Does everyone agree?

S. No, no, no.

T. Julie, read it again.

S. (quickly erases and writes) $J < E$ and read alound Julie's is less than Eric's.

T. It's all right. Julie just got the symbols twisted and has corrected it for us.

S. a. Writes $E > J$

b. Class reads $J < E,\ E > J$

T. Watch as I write this, if $R > E$, and $E > J$, what can you tell me about R and J?

S. a. $R > J$

T. Yes (writes last section)

$R > E,\ E > J \implies R > J$

Let's read it together.

S. $R > E,\ E > J \implies R > J$

T. Which rule is that?

S. The Three Stick Rule.

T. I said ladies first, we talked with Ruth and Julie, now Abby can tell what she sees.

S. Writes on board $P > A,\ A < P$

T. Which rule is that?

S. a. The Two Stick Rule.

b. All read $P > A,\ A < P$

T. Watch as I write this time $R > E,\ E > J$. (We omitted comparing J and P, and J and A.) Now what can you tell me about A's and R's?

S. P > A. I mean R > A.

T. Both are right. Let's add this now and read R > E, E > J
R > A.

S. All read R > E, E > J ⟹ R > A.

T. Can you look at these mountains. How many were we using?

S. Five

T. What shall we call this?

S. Our "Five Mountain Rule".
Intermission ... Class stands.

T. We're going to play another game with this egg carton and our sticks. It's called,"Find My Stick". First I'll hide one stick in this big bag. You are going to try and figure out where it will fit with the egg carton sticks - children watch as T. arranges sticks in this order.

D K G U P O Y N

(D > K > g . . . n)

T. I'll try to stand the sticks in the egg carton. Stick eggs! Notice the size of all the sticks. Let's count them.

S. Class counts eight sticks in carton.

T. You may have 1 clue in Find My Stick Game.
The stick in the bag is r. Look at the carton. You have three guesses to find where the stick in the bag will fit.
Here's my clue: r < d.
You may ask three questions about the other sticks to "Find My Stick".

T. Writes on board
Clue: r < d

S. a. Is r < k?

T. Writes on board - 1. r < k.

S. b. Is r > y?

T. Writes on board - 2. r > y.
You have 1 chance left.

S. (Most emphatically) So, think before you raise your hand!!!

T. Let's read what we know.

S. Class reads $r < d$

1. $r < k$
2. $r > y$

T. Who would like to ask the last question?

S. Is $r > g$?

T. My answer is no.

S. See, you did it wrong!

T. No, she's solving the problem very well. Writes on board -
3. $r > g$ no
Do you know where red will fit Ina?

S. Ina points between g and u.

T. Get the red stick out of the bag and see if that is correct.

S. (Places r between g and u to prove this to the class.)

T. Shall we try "Find My Stick" again?

S. Yes. Yes.

T. Every one close his eyes. I'm going to take all the sticks away to hide another stick in the bag. Ready! Here they are! Egg Sticks!
d, k, g, r, u, p, y, n
"Find My Stick". You have three chances and one clue. O is hiding in the bag.
Clue: $O > n$

S. a. Is $O > u$?

T. I have to check and say no. I'm not marking it wrong. Look 1.
$o > u$ no

S. Is $o > p$?

T. Writes - 2. $O > p$ no

T. Reads to class 0 is not > p.

S. a. Is 0 > y?

T. I'd say 3. 0 > y <u>yes</u>

On board clue: 0 n

1. o > u <u>no</u>
2. o > p <u>no</u>
3. o > y <u>yes</u>

T. Where will o fit with our sticks?

S. a. Comes up and points between purple and yellow.

T. Get the o stick and prove it to the class. (Child does)

S. b. It wasn't > u!
It wasn't > p!
So it <u>has</u> to be > y.

T. How many rules were we using today?

S. Nine - The Nine Stick Rule!

Classroom teacher: Judy M. commented
I don't believe it!

(She was amazed at the strategy and thinking used by her pupils.)

Lexington Public Schools Lexington, Massachusetts

Friday, April 7, 1967 Adams School Above Average - Grade 1

Classroom Teacher: Judy Morecz

Session VII Teacher: B. Fitzgerald

Note: All stick colors were changed prior to the class lesson.
This was done to save class time.
Sticks now are:

Symbol	Length
y	24"
p	20"
o	18"
d	16"
n	14"
k	12"
u	10"
w	9"
r	6"
g	5"

Materials: Sticks taped to chalkboard and two large bags.

Objectives: Use sticks rules studied to figure out solutions in "Find My Stick Game".

Sticks are taped to chalkboard in descending height as:

y p o d k u r g

S. You changed the colors!

T. How can you tell?

S. y wasn't that big!

T. Do you know why I changed them?

S. a. To trick us?

T. Could be!

S. b. To get us mixed up.

c. To learn more.

d. To make us think.

T. What are we going to play today?

S. "Find My Stick"

T. Remember how we played on Tuesday? I'll give you one clue. Then you have three guesses to "Find My Stick."

T. In the bag, I have a n stick. Here's my Clue: p > n.
You have 3 guesses. I'll say yes or no.

S. Is n < d? under Clue p > n

T. Writes on board → 1. n < d yes

S. Is n > k?

T. Yes (writes on board) 2. n > k yes
Where does n belong? (Many, many hands go up).

S. a replies, n > k?

S. b That's just like 2!

S. c We have it on the board already!

T. Writes on board again 3. n > k yes
Show us where n will go. Come up to the board and show us.

S. Comes to board and points between d and k and writes an n between d and k.

T. Get the n stick in the bag and prove it to us.

S. (Get sticks and inserts it properly between d and k.)

Class exclaims its delight and approval -

Game 2

T. This time w is in the other bag.
My Clue is : p > w.

S. Is w > r?

T. (Writes on board) 1. w > r yes.

S. Is w > u?

T. (Teacher asks class) What am I going to say?

S. No! No! No! (in unison)

T. (Writes on board) 2. w > u no

T. Who's ready to "Find My Stick"? -------- guess three?

S. u > w.

T. Do you agree?

S. Chorus - "Yes"

T. Writes on board 3. u > w
Can you place the stick symbol on the board correctly before you get the stick from the bag?

S. a. Writes - w between u and r.

S. b. Class is elated - "Hurray!"

T. After our vacation, the next time, we will play with our sticks and be thieves and detectives.

NOTE: Class enthusiasm and response was great. This lesson was completed in 9 mins. There are twenty class members present.

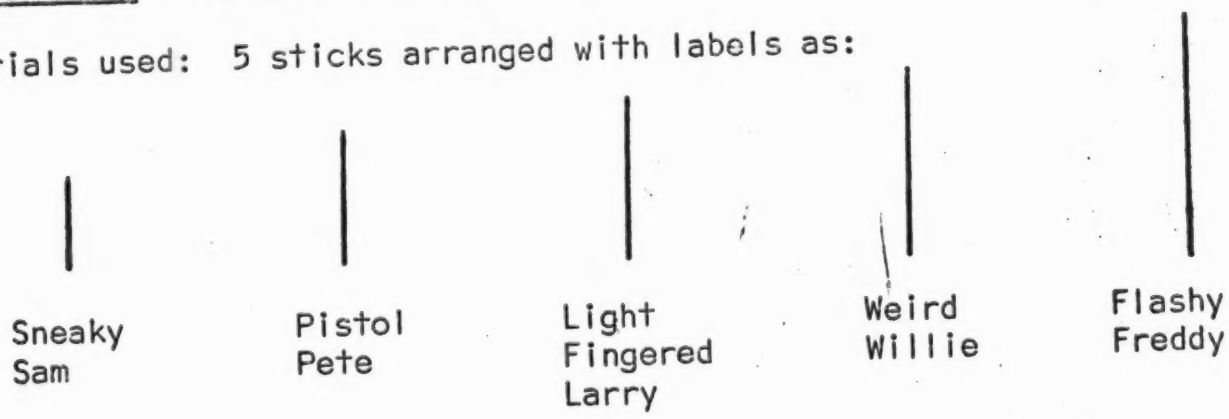
LEXINGTON PUBLIC SCHOOLS Lexington, Massachusetts

Tuesday, April 25, 1967 Adams School Above Average - Grade 1

Classroom Teacher: Judy Morecz

Session VIII - Teacher: Billie Fitzgerald

Materials used: 5 sticks arranged with labels as:



S. You said we'd play robbers!

T. We are. Look at the blackboard. Let's read all the names together. (Class reads in unison.) Now I'd like to tell you about these men. We're going to make believe they're a gang of robbers. See them lined up here at the police station. (points to board) All of us will be detectives and help the police look for the stolen diamonds these robbers stole. In this chair will be the master criminal, their leader. His name is Dead Eye Dick. He knows who has the diamonds. So the police have brought the gang with Dead Eye Dick and put him in this chair. The police have a machine called . . .

S. An electric chair!

T. No. It's a lie detector. The police put the wires on Dead Eye Dick's arms and he answers our questions. We have three search warrants to ask three questions about the diamonds. You're the detectives and you ask if the robber with the diamonds is taller or shorter than another robber. Whom do you think has the stolen diamonds?

S. Is he Weird Willie?

T. No. (I'm answering for Dead Eye Dick as I read the answers on this lie detector machine. Which robber can we remove from the line-up?

S. Just erase Flashy Freddy's name.

T. O.K. Flashy Freddy is out of the line up.

S. Is he shorter than Pistol Pete?

T. No. He is not shorter than Pistol Pete. Whom do we eliminate now?

S. Sneaky Sam.

T. One more chance! Here's your last search warrant.

S. Is he taller than Pistol Pete?

T. Yes. Ruth, who is it?

S. It's Light Fingered Larry. I <u>knew</u> it.

T. Question?

S. Can one of us be Dead Eye Dick?

T. Good. Let's have James be Dead Eye Dick first. (James sits in chair with "lie detector strapped to him." Children asked to have his name, "Dead Eye Dick" written above James on the black-board. T. and James confer and James decided on diamond location.)

T. This time I'll help James. Next time he'll answer. These robbers are very foxy. They may pass the diamonds from one to another. We now have three search warrants.

S. Is he taller than Light Fingered Larry?

T. Yes he's taller than Light Fingered Larry.

T. Do you know which robbers we should eliminate from this line-up? (Children mention first two stick names.)

S. Is he shorter than Weird Willie?

T. No. Can you tell me who it is?

S. Weird Willie?

T. No

S. (Several in unison.) Flashy Freddy.

T. Yes.

T. This time James will answer all your questions.
(T. and James confer as James designates diamond location.)

T. Let's use our first search warrant.

D.= Dead Eye Dick (James) calls on pupils.

S. Is he shorter than Weird Willie?
D. Yes another search warrant.
S. Is he shorter than Light Fingered Larry?
D. Yes.
S. Is he taller than Sneaky Sam?
D. No. You've had three search warrants!
S. It's Sneaky Sam!
D. That's a hard job being Dead Eye Dick! But I was prepared for anything!
T. Good for you Dead Eye Dick!
(Anne whispers to T. the selected robber and sits in chair. Anne also = "D".)
S. Is he taller than Light Fingered Larry?
D. Yes.
S. Is he shorter than Flashy Freddy?
D. No.
S. It's Flashy Freddy!
D. You're right!

T. Today is Sandy's birthday. Let's play it once more and Sandy may be Dead Eye Dick. (Confer with T.)
S. Is he shorter than Flashy Freddy?
D. (He gulped and was astounded.) Nope! (Note: Immediately many children realized where the diamonds were located after the first question. The few that were not paying close attention, continued to ask questions.)

S. Is he taller than Light Fingered Larry?

D. Yes.

S. Is it Weird Willie?

D. No

S. It's Flashy Freddy. (and most of the class was ready for this correct answer.)

T. What does what we were doing today remind you of?

S. The stick rules we've been playing.

Lexington Public Schools Lexington, Massachusetts

Tuesday, May 2, 1967 Adams School Above Average - Grade 1

Classroom Teacher: Judy Morecz

Session IX Teacher: Billie Fitzgerald

S. Where's Dead Eye Dick today?

T. Dead Eye Dick is in jail. He stole too many diamonds. Today we'll play another game called Rocks, Paper, and Scissors.

S. There's a game like that that I know.

T. What is it?

S. If you have scissors and rocks, rocks win because they can break scissors. If you put paper and scissors out, scissors wins because they cut paper.

T. That's what we're going to play. Where did you learn this game?

S. a. My brothers taught me.
b. My sister played that with me.

T. Let's have Kathy and Sandy show us how they played the game after we read these symbols on the board.
(Class reads "R" as T. explains "R" means rock and is shown by doubling first; "P" means paper and is represented as an extended hand; "S" means scissors and is represented by extending the first two finger open.
Entire class practices as T. calls signals.)

S. Sandy and Kathy demonstrate at front of class.

T. The "R" wins over the scissors. Why?

S. Because the rock breaks the scissors.

T. (Writing at blackboard) So we can say the R wins over the S and write it as: R > S. What about the S and P?

S. S wins over the P because it can cut up paper.

T. (writes) S > P. (class reads with > "wins over") Now what about the paper and the rock?

S. The paper will wrap the rock so P > R.

T. (Class reads from blackboard)

R > S
S > P
P > R

T. This time as Sandy and Kathy play we'll keep score. (Children stand in front of room to demonstrate.) Notice how they put their hands behind their backs.
Ready when I say "1,2,3" -

S. No! Say, "1,2,3, shoot"!

T. When I say shoot they will bring their hands out using one of our signals. 1,2,3 shoot.

S. Sandy won!

T. Why did Sandy win?

S. R wins over S.

T. (Points to blackboard R > S) and places tally under score as:

K	S
11	1

Are you ready again?
1,2,3, shoot. Who wins?

S. Kathy wins again!

T. Why?

S. P > R.

T. Ready. The score is now tied. 1,2,3 shoot.
It's a tie. (both had R) No score.
Try again. 1,2,3, shoot. Who won?

S. Kathy because S > P.

T. Look at the score 2 - 1: Kathy won this time.
Let's have Patty and Jeff come up front this time. 1,2,3, shoot.
Who won?

S. Jeff. R > S.

T. 1,2,3. shoot. Who won?

S. Patty. Patty put out a rock.

T. Once more. 1,2,3 shoot

S. Jeff wins! P > R.
(Lesley kept tally score on board as children played.)

T. Now let's all play. (Check to see that everyone has a partner; children play independently in pairs).

All S. 1,2,3, shoot! (Individual winners clap, then continue game.)

T. Let's have just our winners stand. Let's all go back to our own seats. Look at the board. Let's read this again all together:
R > S, S > P, P > R.
Now I'm going to use something else. Can you guess?

S. Oh! sticks.

T. One is going to be the rock (p) and one is going to be the scissors (d). What can you tell me about R + S?

S. R > S The R breaks the S.

T. You're using sticks this time.

S. p > g (T. writes on board and S. read.)

T. I need something else here - (points to right at board) What color?

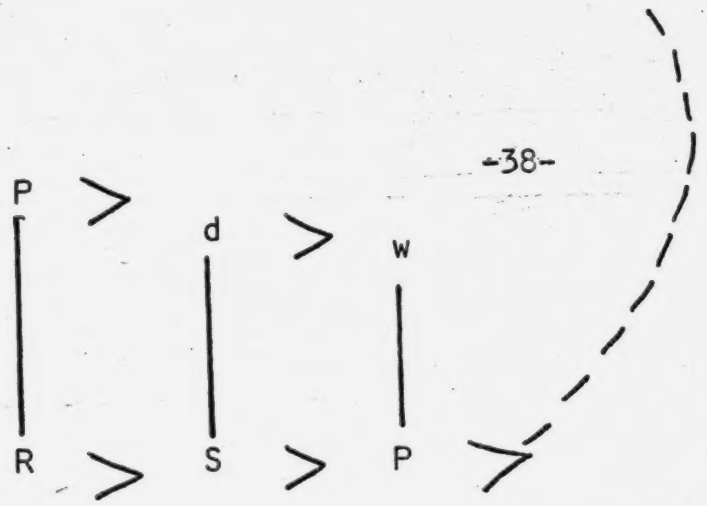
S. White because paper is usually w.

T. What can you tell me now?

S. g > w.

T. Let's read this (points to first recording that was written at beginning of class - P > R) What happens now?

S. We can put > here. (after Pat right)
Diagram of sticks on board as each part is developed



S. a. No.
b. I don't get it.
c. (Child at board add > then goes to R and put -- in front.)

T. Is P > R?

S. No (erases > at right and line in front of R)

T. It says here P > R. What do you think? Is the paper bigger than the rock?

S. On the blackboard, the rock is bigger with the sticks.

T. Exactly right. What happened to your 3 stick rule?

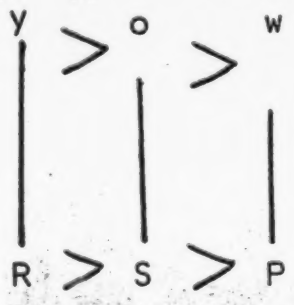
S. It didn't work.

T. Do you remember what I told you a long time ago?

S. It might not always work.

T. Do you think it might work with some other sticks?

S. MAYBE! (Class)
(T. and a few S. select new sticks and record on board)



S. a. P is a <u>lot</u> smaller than the rock now.
b. The sticks work.

T. What did you learn from today's lesson?

S. How to play R,S,P.

T. And what did you learn about the 3 stick rule?

S. It doesn't always work.

Note: At the beginning of the next session, check with class - did the 3 stick rule work for R,S,P, and did the 3 stick rule work for the 3 sticks at the board?

Lexington Public Schools
Lexington, Massachusetts

Adams School
Above average - Gr. 1

Tuesday, May 9, 1967

Classroom Teacher: Judy Morecz

Session X Teacher: Billie Fitzgerald

T. We've been playing stick games. Can you remember the game we played last time?

S. Rock, Scissors, Paper.

T. Did the Three Stick Rule work?

S. No.

T. How did you know?

S. We lined up the sticks for Rock, Scissors, Paper ... and it didn't.

T. Today we'll use the same inequality symbols greater than and less than. I'd like to see if you can use the symbols in adding the sticks. Who can come up and try? Any one have any ideas how we'll add stick using < and > ?

S. (Linda volunteers. She selects g,r, w and lines them upright in the chalk tray observing her arrangement.)

T. What will you do to add the sticks? Think of how you add. Now can we add these sticks?

S. Several start to suggest to Linda as,

a. I know
b. Take 1 and add another
c. You can take 1 or put the others together.
d. Julie comes up, "I can put them together."

T. Let's try. I'll take one (w) and you add one of your sticks to it.

S. (Takes r and put it parallel to w.)

T. What do you know about these sticks?

S. Writes at board: w > r

T. Now take your other stick.

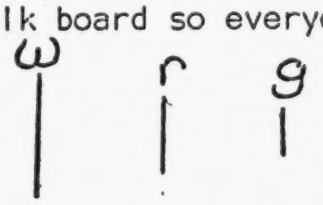
S. Writes at board: r > g

T. Now can you finish this by using the Three Stick Rule?

S. $w > r,\ r > g \Longrightarrow w > g$
(others clap for Julie)

T. Let's read the Three Stick Rule together. ($w > r,\ r > g \Longrightarrow w > g$)

T. Now we'll try to add the sticks. I'll draw this diagram on the chalk board so everyone can see. (relative sizes were similar)



S. Holding r stick puts g beside it, and says "$g < r$."

T. Can we add w to the sticks? (Writes "g + w" and places sticks on desk end to end at edge of desk. Records length with chalk mark.)

S. Now add "r + w" (Repeats placing sticks end to end and makes chalk mark on desk.)

S. (All gather around the desk to watch stick placement.)

T. Which is bigger? How can we write it at the board? Use our inequality symbol.

S. Writes $g + w < r + w$

S. All read together "$g + w < r + w$."

T. Kathy, you said you had another way.

S. Take r + w. Put w over r and you get pink!

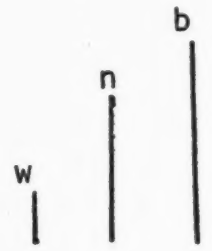
T. Good Kathy, That's right but I'd like you to add and measure lengths of sticks.

S. (Linda comes up again. More positive this time) Add g+r. (She goes to board and writes $g < r,\ r < w \Longrightarrow w > g$.

T. Is she right?

S. a. Yes.
b She turned the rule around.

T. Use some of these sticks to add. (Tapes them to board so class can observe sizes.)



What can you tell me now?

S. Writes and says, "w < u."

T. (Removes sticks from board and makes line diagram.) Can some one take the brown stick and add it to the others?

S. a. Takes and puts w + n end to end, placing them at edge of desk and marks length on desk with chalk.
b. Same procedure with n and b measures
c. All observe chalk marks
d. Writes: $w + n < b + n$

T. Can some one choose other sticks?

S. I want r, k, d.

T. We'll add these sticks. (Tapes to board.)

What can you tell me about k and d?

S. $k < d$

T. Can we write this on the board as we add now?

S. r + k

T. What next?

S. r + d (written on board as r + k r + d)

T. What inequality symbol can we use? Let's measure on the desk.

S. a. Measures r + k then chalk marks desk.

b. Measures r + d then chalk marks desk
c. Writes " $<$ "
d. All go to look at sticks and chalk marks on the desk.

T. Let's read this " r + k $<$ r + d."

S. Read, "r + k $<$ r + d."

T. We'll add 4 sticks this time. (child gives T. u, w, r, g)
Tapes to board as: u

What can you tell me about u and w?

S. Writes "u $>$ w."

T. What about r and g?

S. Writes "r $>$ g."

T. Let's look at what we know:

u $>$ w
r $>$ g
Can we add u + r and w + g?

S. a. Takes u + r and places them on the desk. Notes edge of desk with edge of stick and makes chalk mark at the top.
b. Does same with w + g."
c. Goes to board and puts in symbol u + r $>$ w + g.

T. Good for you! Let's all read this now.
Let's add the sticks another way. Look at u and r. What can you tell me?

S. u $>$ r

T. Now what can you tell me about w and g?

S. a. w $>$ g

T. Let's add down this time. Points. (at board, u $>$ r
w $>$ g)

S. Abby takes and add sticks, u + w and records with chalk mark on desk and writes on board "u + w." Does the same with r + g.

T. Can you finish this now by putting in the inequality symbol?

S. $u + w > r + g$

T. Who can tell us about u and g?

S. a. $u > g$ (Writes on the board)

T. What about w + r?

S. $w > r$ (Writes on board $w > r$)

T. Can some one add these and finish this on the board?

S. (Measures u + g and records on desk, with chalk. Does the same with w + r.) Writes at board "u + g w + r" and class notes markings on desk and reads.

S. b. My idea was I could use the Three Stick Rule backwards using less than.

S. c. I'm trying to figure if r + g will equal another stick that we have.
(Child measures r + g with numerous sticks. It doesn't work. Holds sticks in fists as he works.)

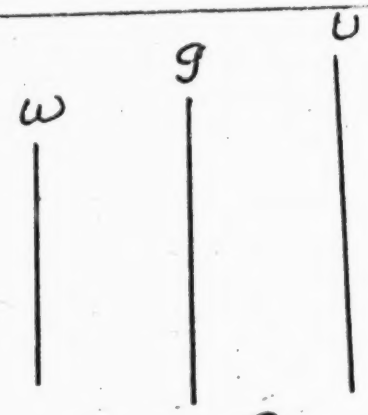
T. Is it possible?

S. We don't have any sticks the right size to make it work.

T Let's see if we can do this work sheet. You are to fill in the placeholders and blank lines with the correct letter symbols.

LESSON 10

A.



Three stick rule

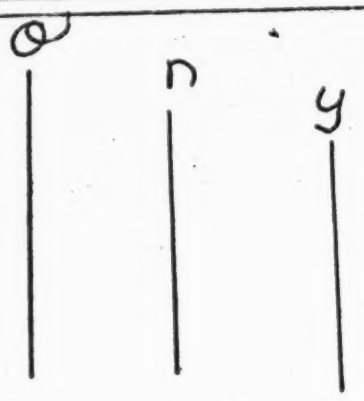
~~$w < g, g < u, \Rightarrow$~~ □ < △

$u > g$

$g > w$

$w + u >$ ——

$u + g >$ ——

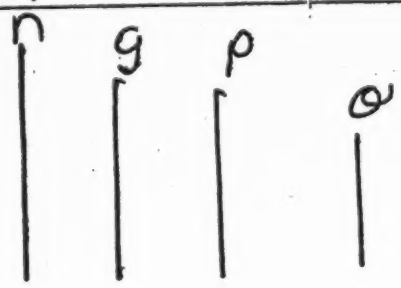


Three stick rule

$\omega > n,$ —— $\Rightarrow$ ——

B. $\omega > n$

$\omega +$ —— $>$ $n + y$



C.

$r > g$

$p >$ ——

$r + p > g + \omega$

$r > p$

$g > \omega$

$r + g >$ $p +$ ——

$r > \omega$

$g > p$

$r + g >$ $\omega +$ ——

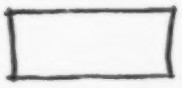
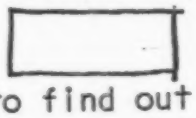
LEXINGTON PUBLIC SCHOOLS
Lexington, Massachusetts

May 31, 1967
Adams School

Tuesday, May 16, 1967

Classroom Teacher: Judy Morecz

Session XI Teacher: Dr. Lomon

T. I've changed my business. I'm no longer in the mountain business. I'm a carpenter now. I have some special problems and I hope you can help me. I need different sizes of wood.
A customer has a step on his porch with certain size board. 
(draw illustration on board).
In my shop, I have a lot of boards. Sometimes the customers don't want the same size board. If a part rots or is broken off, only a piece of it needs to be replaced. We can't have a hole in a step as someone may fall. So the customer may ask for a scrap of wood. The customer brings  piece and the broken piece. How are we going to find a good piece to fit? (Holds 2 sticks). We have to find out if it's the right size.

S. Measure the good piece you have and measure the rotten piece he has.

T. Good. Is there a simpler way to measure a replacement?

S. It's just a little bit too long. They are not compared yet.

T. O.K. You measure them. Mark the good piece in the shop g and the rotten piece r.
(children later change r to o because of colored chalk being used.)
What will happen if I try to add g to the big piece he has left on his step. Can you mark it down so we can see which is bigger.

S. Writes on board: $g > o$.

T. Let's use p for the good step at home.
(Marks on board: step $\frac{P}{\text{from home}}$ $\frac{O}{\text{rotten piece}}$)
This is what he brought. Can he take g to add to step?

S. It's too big.

T. Who can write that down?

S. Writes: $g > p$

T. How can we add the sticks?

S. a. P ? (hesitates)
b. p + g > bigger than what?

c. $> p + o$
d. Says: $p + g > p + o$

T. Compare: $g + o \quad p + o$
Is this right $g > o$

$p+g > p + o$
Who can check it for me at the board? Show me.

S. Abby puts p + g sticks at board. Then p + o sticks.

T. Some one mark where the g stick ends ... then the o stick. I'm going into my shop and find another piece of wood. (Holds U in hand. It's smaller.) Can I use the u stick to replace the o stick?

S. (Holds u + o) u o :

T. Is this correct?

S. Changes o + u.

T. Is that how I try to check? How can we check?

S. (Checks sticks by putting together.)

T. Will it do?

S. (raise hands) No. No.

T. Why?

S. a. It isn't as long as blue.
b. U isn't as long as o.

T. Right. O is bigger. If the customer took U home to attach to p, what would happen?

S. Too small!

T. Can you write it?

S. a. Writes: $U < O$
b. Two other check U + O and mark at board.

T. If $U < O$ then what?

S. $p + u < p + o$

T. What else do you know about in equalities and other rules?

S. a. Take rotten one and one bigger and saw it.
Then you'll have the right size.
b. They are both three stick rules.

T. This is Kathy's three stick rule.

T. A man comes into my shop. His step is this size (p). He says "I couldn't find the rotten piece but this is left. I only need a piece to put with what's left?
How do we find what the man needs? _____ _____ (left over piece-good one)

S. All you have to do is look where the boards are and find one that equals this (as he marks sticks with chalk)

T. Will customer be satisfied or not?

S. Satisfied!

T. Is that a way to do it?

S. Yes. (all agree)

T. This stick is the step. (p) O is good piece from the step. How big a piece must I make?

S. Same way Peter just did it.

T. You do it.

S. Unless it's the same size.
(lines up sticks and mark off top of o with chalk on board)

T. Will the customer be satisfied?

S. Yes.

T. Did Peter and Eric add sticks?

S. Yes.

T. Someone come up and show how we add p + o.

S. (lines sticks on board end to end as: p o and marks one end)
Writes: p + o

T. He says p + o. That's right. But the piece we had to give him wasn't that long.
Is that the same as p + o?

S. No.

T. Is it > or < ?

S. It's bigger but all you do is saw it!

T. Is Peter's rule adding?

S. a. Kind of like comparing.
b. Why don't you just give him the whole piece?

T. Too much money.

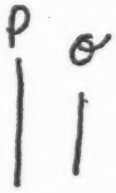
T. Peter, when you had to find out what piece to add to make it come out to the first stick, did you add sticks?

S. See if you could add on to it?

T. He saw what he'd have to add.

S. To find a piece this small.

T. What is 2+3 = ☐ ?
What is 2= ☐ = 5?

Is this example in numbers the same as p + o or like  ?
Which sticks are like this?

S. The first.

T. Which is this 2 + ☐ = 5? Is that like the other one?

S. They are both sort of like the same.

T. What do you do to 2 to get 5?

S. a. We add.
b. We're looking for something to add.

T. If I had $10 and I gave away $7, what would I have left?

S. $3.

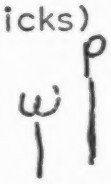
T. How did you get that?

S. Subtract.

T. Can you find the answer for 2+ ☐ =5 by doing a subtraction problem?

S. (No answer)

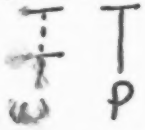
T. (Using sticks) Can someone subtract p + w by doing what Peter did?

S. Compares 

T. Subtract w stick from p stick and see what's left over.
Show me on the board.

S. Arranges sticks as:

T. Draw a heavy line over piece left over. Can you write it down?
She had a purple stick and subtracted w. What's left over?
Write it on the board. Who can write it?

S. a. shows 

(: = ø line drawn

b. Writes: p - w = o

T. Let's read what she has written.

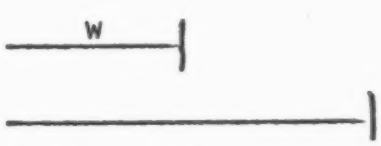
S. All read p - w = o.

T. Let's see if we can mark it off on the board so we can see how we added the stick.

S.

(to add logs sticks on board horizontally)

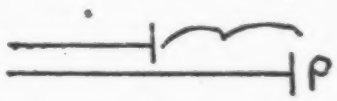
T. Who can come and draw and mark over left over part?

S. 

(procedes to mark lengths of p, then w)

T. What's left?

S. P-W

T. Adds to board diagram what's left as: 

This is what's left . It's what we get when we subtract w from p. Kathy mark in what's left with colored chalk.

S. (Kathy marks in U with colored chalk.) Writes on board p-w = u

T. Take a big stick.. Look at U. Cut it off. Give it to the customer so he can repair his step.

LEXINGTON PUBLIC SCHOOLS — Lexington, Massachusetts

Tuesday, May 23, 1967 — Above Average - Grade 1

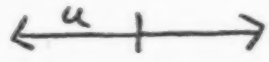
Classroom Teacher: Judy Morecz

Session XII - Teacher: Billie Fitzgerald

T. At our last class, Dr. Lemon told us about his carpenter shop and that some of his customers needed lumber to repair their broken steps. Well, his shop had a big fire. The fire is now being investigated.

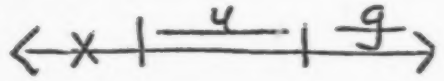
S. What's that?

T. Searching for causes of the fire. Quite a bit of his lumber was damaged. Now he's shopping for new supplies.
I'm opening a little shop with some lumber scraps that were saved from the fire. I'll take care of his customers for a little while.
Today we'll begin with a stick line on the board instead of using a desk. (⟷)
Let's add u and g and g+k.

S. Marks on board 

T. What else do we need?

S. We have to put down the other side or else you won't know where to begin.

S. Marks 

T. Can you write what you've done?

S. Writes on board: u+g

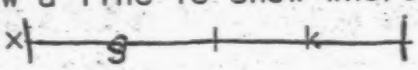
T. Now let's add g + k on the stick line.

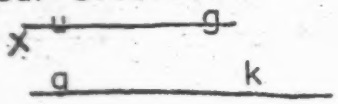
S. a. (places g again next to g on stick line)
b. (two pupils holding ends of stick)

T. Is this correct?

S. a. No
b. They need to fix it and put it on the left side of the stick line.

T. Right. Draw a line to show where u ends. Then label your sticks.

S. a. Draws 

so stick line now is: 

b. Writes to the right g+k.

T. Look at our stick line now. Who can finish writing about it?

S. a. (add symbol $<$) Says: $u + g < g + k$
Erases $<$ and changes symbol to $\Longrightarrow$
b. It should be "is less than"
c. Changes back to $<$

T. What did we find out when we added 3 sticks?

S. $u + g < g + k$

T. Let's all read this. Do you agree?

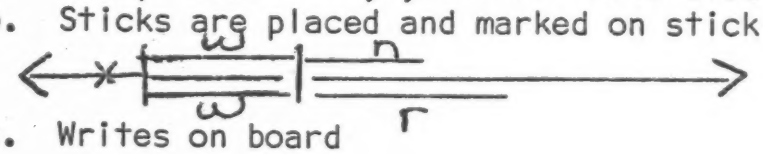
S. Yes.

T. When we add, what did we do with the sticks.

S. a. Put them together.
b. Put them end to end.

T. Now Ina and Eric will add 3 other sticks using our stick rule.

S. a. (They select: w,r, and n) Hold sticks at board.
b. Sticks are placed and marked on stick line as:



c. Writes on board

T. How many agree? (hands are raised)
What did we do?

S. Put them together end to end.

T. What operation were we using?

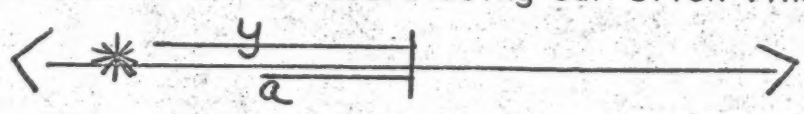
S. Adding.

T. Now let's try this. Peter comes to my store.
This is the size of his step. (y)
He brought me the good piece of his step (u). How can we find out how to get the rest of his step?

S. Hold the sticks side by side.

T. Can you show me on the board using our stick line?

S. Draws:



T. What are we doing with the sticks this time?

S. Subtracting!

T. Who can write what we just did?

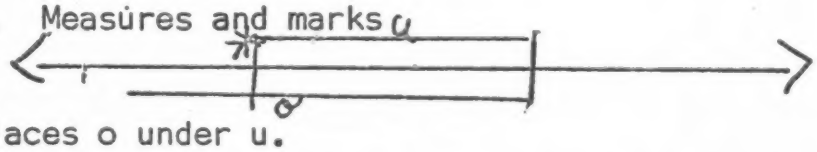
S. Writes y-u

T. Who can show us the size of the piece this customer must have?

S. Draws on stick line as:

T. This time the step size is u. The customer gets this size piece of wood at the lumber yard o. Who can put this on our stick line?

S. a. Measures and marks u



Places o under u.

T. What happened?

S. It's too long.

T. Let's mark it to show how much too long.

S. Marks as

T. What happened to our sticks?

S. They were too long from the starting point.

T. What operation were we using?

S. Subtraction.

LEXINGTON PUBLIC SCHOOLS

Lexington, Massachusetts
Above Average - Grade 1

Tuesday, June 6, 1967

Classroom teacher: Judy Morecz

Session XIII - Teacher: Billie Fitzgerald

T: What am I drawing on the board?

S. a. Number line?
b. Stick line?

T. Why?

S. Looking at it you can tell if it's either one.

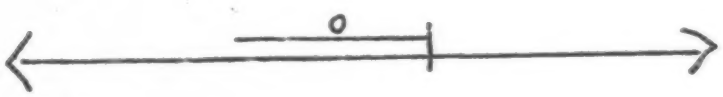
T. How can we use it?

S. a. Measuring sticks.
b. Counting with numbers.
c. Make it a number line.

T. We'll need help to make a number line. Let's make it a stick line and a number line. Can some one add some sticks using this line?

S. (Selects w and o and g.)

T. Show us how to add these sticks.

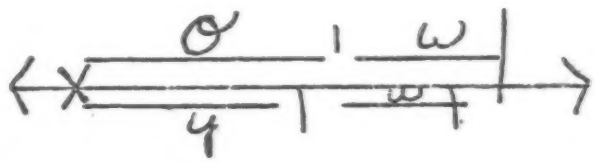
S. 

T. What is missing?

S. We need a mark to start at.

T. How can we do it?

S. Marks at board (x)



Then marks g and w.

T. Can you write what you found out?

S. Writes: $g+w < o+w$

T. Could I say this? (writes on board)
$w+g < w+o$

S. a. Yes.
b. It's just backwards.

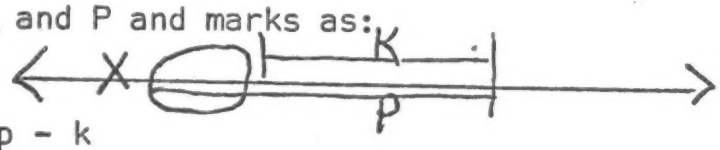
c. If it's backwards it could be (comes and writes on board)
o+w > g+w
It means that w+g is the opposite of g+w.
(Abby uses diagram to explain as she talks)

T. I'd like to have 2 boys show us another operation on the stick line. What is it?

S. Subtracting.

T. Choose two of our sticks.

S. (Gets K and P and marks as:



Writes p - k

T. Who can show me the difference.

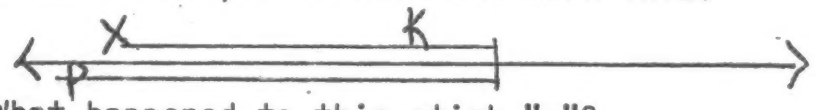
S. Draws "circle" as shown above.

T. We did p-k. If I begin with k - may I take p away?

S. a. yes
b. no
c. yes, if you put a negative.

T. Absolutely right. How would it work on our line?

S. I need a helper to hold and mark this.

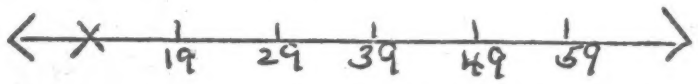


T. What happened to this stick "p"?

S. Negative number.

T. The starting point is important. Let's try something else. Can anyone add this stick (g) to itself?

S. a. Comes up and shows and marks



b. Class names each point as labelled.

T. What can we do now?

S. a. Erase the "g's"
b. Oh, oh. Now it's a number line.

T. What unit of measure did we use to make it?

S. A plain green stick.

T. As I look at that number line, I see something else you can do.

S. a. I know. (goes up and writes positive symbol in front of every number.)

T. Good. What else were you thinking?

S. a. Starts to erase positive symbols saying, "I wanted to make it look like a ruler."

T. Let's try something else right now. What else is missing?

S. Negatives.

T. What else?

S. a. Zero
b. You always have to have a zero.
c. You can now erase x if it's a starting point.

T. (Adds o to replace x) What else can we do?

S. Put negatives on.

T. We'll use g as a unit of measure. (adds large paper to supplement board as class is working at end of chalkboard)

S. Holds g and marks negative points to end of paper.

T. Who can do something else to this?

S. Labels -1, -2, -3 ... to end of line on paper.

T. Could we make another number line using "k" as the unit of measure?

S. Yes. You wouldn't have much room though! (k is larger)

T. Would we have as many points?

S. No. (holds k up on number line k is about twice as big as g). It looks like we're counting by 2's when you use k.

T. Can you figure my problem using our number line?
Mr. Geick, our principal asked Miss Morecz to give packages of paper to the first grade teachers on this floor. Miss Morecz took 4

packages for her room, and 7 packages for Miss Allton's room. How many packages of paper would Miss Morecz need?

S. Eleven.

T. What can we do with our number line?

S. a. Put things on it.
b. Add.
c. Write numbers on it.
d. Write positive and negative.

LEXINGTON PUBLIC SCHOOLS — Lexington, Mass.

Tuesday, June 13, 1967 — Above Average - Gr. 1

Classroom Teacher - Judy Morecz

Session XIV - Teacher: Billie Fitzgerald

T. Today we're going to play a Postman game. The Postman carries only bills and checks in his mail bag. He delivers them to the people on our street. Can you think of a name for him?

S. Postman Bill

T. Good.

S. Call him Bill because he delivers bills.

T. Would you be happy or sad if Mr. Bill gives you a check?

S. Happy.

T. What's a check? (Takes one from pocketbook) I have one here. How much money will I get? (Children all look at check.)

S. $23.25.

T. I haven't had time to go to the bank, and cash it. Would you be happy or sad with a bill?

S. Sad

a. My dad has millions of them.
b. Bills! Bills! Bills!
c. We have bills.

T. Mr. Bill often has so much mail that he gets mixed up and leaves the bills and checks at the wrong houses. When he realizes his mistakes, he picks up that mail.

S. A. That's our mailman!
b. Ours does that, too.

T. Then he has to pick it up again and take it to the right place. If Mr. Bill brings a check for $5.00 are you richer or poorer?

S. Richer.

T. By how much?

S. $5.00

T. Draws at chalkboard and records:

brings check

If Mr. Bill brings you a bill for $3.00· are you richer or poorer?

S. Poorer.

T. By how much?

S. $3.

T. Draws at chalkboard and records:

brings bill

If the postman comes and takes a check away from you and says: "I'm terribly sorry but that check for $8 isn't yours. Are you richer or poorer?

S. Poorer

T. By how much?

S. $8.

T. Draws at chalkboard and records:

Takes check away

Now if Mr. Bill comes and takes away a $7 bill away from you and says, I'm terribly sorry that isn't your bill. Are you richer or poorer?

S. a. Richer.
b. We're glad.

T. By how much?

S. $7.

T. Draws at chalkboard and records:

takes bill away

Let's look at all four faces at the board and read what has happened each time. Now let's pretend it's Monday morning. Mr. Bill is coming and he rings the bell at Jeff's house. Brrring! Here's a bill for $9 (Jeff cries out) and a check for $6.

Are you richer or poorer? (Teacher has 2 oak tag cards labeled $9 and $6.)

S. Poorer by $3.

T. Next day, Tuesday, Mr. Bill goes to Sandy's house. He gives you a check for $12 and a bill for $7. Are you richer or poorer?

S. Poorer.

T. Repeats showing 2 oak tag cards labeled $12 and $7.

S. Richer by $5.

T. On Wednesday, Mr. Bill goes to Ira's house and gives her a bill for $6 and a bill for $1 (use oak tag cards). Are you richer or poorer?

S. Poorer by $7.

T. Mr. Postman has a busy week. On Thursday, he gives Mark a check for $6 and takes away a check for $5 that he had left before. Is Mark richer or poorer?

S. Poorer by $5.

T. Think again. (shows oak tag cards and reviews Thursday's mail)

S. Richer by $1.

T. Yes. $1 is right (xplains problem to class again.)

S. Oh! 6-5 = 1. Just a dollar.

T On Friday, Mr. Postman comes to this house. He gives you a check for $15 and takes away a bill you already had for $3. He said, "Sorry, this bill doesn't belong to you." Now are you richer or poorer?

S. a. $15?
b. $ 3?

T. Watch again. It's tricky. (Repeats whole story using oak tag cards for $15 and $3).

S. Richer.

T. How much richer?

S. a. $10?
b. $12?
c. $ 5?
d. $15?

T. I don't think you're listening carefully. I'll explain Friday's mail. (Repeats showing cards.)

S. $18 richer.

T. Yes, it's a check for $15 and took away a bill for $3. So you'd be $18 richer. On Saturday, Mr. Bill is at Jame's house. He gives you a bill for $5 and takes away a check for $7. Are you richer or poorer? (Shows oak tag cards)

S. a. $7
b. $2 poorer
c. Negative two
d. $12 poorer. It's a bill for $5 and takes away a check for $7.

T. This time let's do it a little differently - We'll keep track of what Mr. Bill does. He has so much mail.
Supposing it's Monday and he's at Julie's. He brings a check for $10 and a bill for $4. (Shows oak tag cards.) Is Julie richer or poorer?

S. Richer by $14.

T. No. He brings a check for $10 and a bill for $4.

S. $6 richer.

T. Let's write this story. When Mr. Bill brings a check (refer to diagrams made at beginning of class) we'll use a positive symbol. When he brings a bill, we'll use a negative symbol. Writes and refers to Monday story:

$$^{+}10 + ^{-}4 = ^{+}6$$

S. All class reads equation as written by teacher.

T. On Tuesday, Mr. Bill brings a bill for $9 and takes away a check for $9 you had.
Are you richer or poorer?

S. Poorer by $18.

T. Can you tell me what to write on the board? All bills have negative symbols.

S. Says $^{-}9 - ^{+}9 = ^{-}18$. Very poor!

T. On Wednesday, Mr. Bill gives you a check for $4 and a bill for $4. Are you richer or poorer?

S. He's medium!

T. Right!

S. If a man gives a $4 check and a $4 bill - it's the same.

T. Yes. A check for $4 and a bill for $4. Jeff was right. What do I write next?

S. Negative four, no, positive four first.

T. Writes: $^{+}4 + {}^{-}4 =$ ____ What happens when we add?

S. Zero!

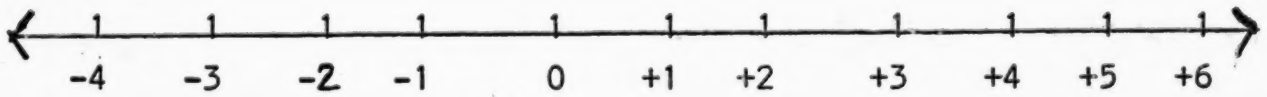
T. On Thursday, Mr. Bill goes over to this house. He gives a check for $2 and a check for $3 then he gives a bill for $10. Richer or poorer?

S. Poorer. Negative 5.

T. I'll write it like this:
$(^{+}2 + {}^{+}3) + {}^{-}10 =$ ____
What was that answer?

S. Negative 5!

T. Let's use our stick number line for Mr. Bill's work on Friday - Remember how we used a stick unit to measure last week. Today I'll use this unit of measure (2" strip of oak tag). I'll put points on here to make my number line. You say it for me. Where do we begin?



S. Zero, positive one ...

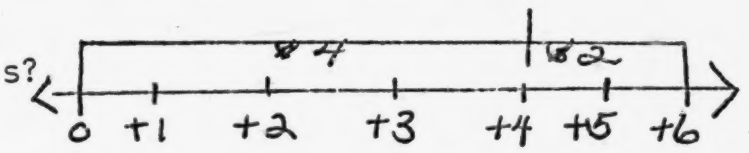
T. Let's do the other side.

S. Negative one ... continued

T. We'll show on the number line what happens as Mr. Bill delivers the mail. He gives us a check for $4 and a check for $2. (Holds oak tag strip with $4 on it measuring 8 inches. Then an oak tag strip with $2 on it measuring 4 inches.)
Can some one show us on the number line and write the equation?

S. Places $4 card on number line and adds $2 card at right.

T. Can some one write an equation for this?



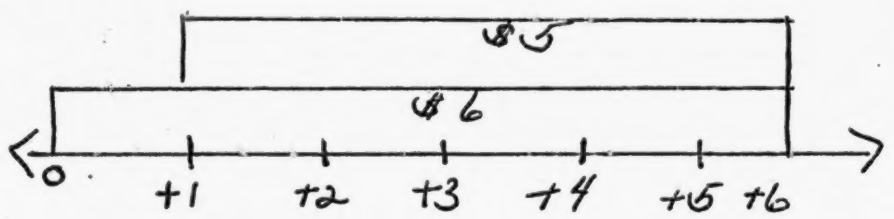
S. Writes: $^{+}4 + {}^{+}2 = {}^{+}6$

T. Let's read the equation together. Can we do another for Saturday? Mr. Bill brings a bill for $5 and a check for $6. Richer or poorer? (Holds oak tag strips.)

S. Puts $6 on number line correctly. Hesitates.

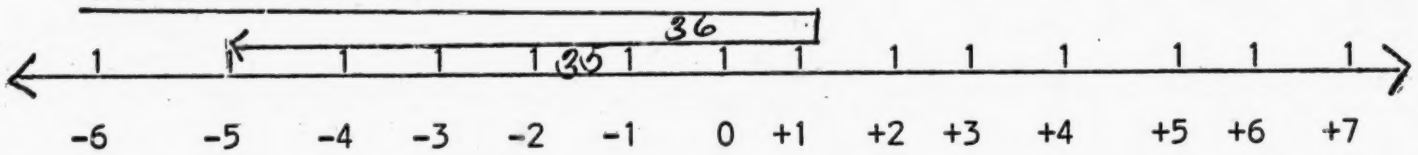
T. Now it's a bill for $1.

S. Places on line as:



T. How much poorer? Can you write it?

S. Writes on board $^{+}6 + {}^{-}5 = {}^{+}1$ (should have written $^{-}5 + {}^{+}6 = {}^{+}1$)
This should have been:



T. Mr. Bill is going home now.

LEXINGTON PUBLIC SCHOOLS — Lexington, Massachusetts

Tuesday, June 20, 1967 — Above Average First Grade

Classroom teacher - Judy Morecz

Session XV Teacher - Dr. Lomon

Materials: Worksheet - sample enclosed and strips of colored paper
1 red one 1/2" x 2¼" and 2 blue 1/2" x 1-1/2"

T. I've heard you've done all sorts of adding and subtracting. We'll play a game with the papers just passed to you. The one who wins will add and subtract fast.
Why is there an X on your paper?

S. It's the starting point.

T. Yes. If you add sticks what do you do?

S. Put them end to end.

T. If you put them on starting point, where do you go?

S. To the right.

T. What happens if you subtract.

S. Go to the left.

T. You have 2 sticks. Which are bigger?

S. The red sticks. (Holds up red paper.)

T. Who can write it?

S. Writes: red > blue.

T. Is that right?

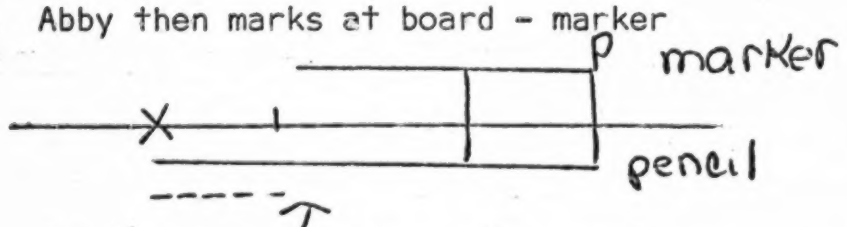
S. Yes!

T. When I say go, tell which "blob" it ends in or the nearest. (Writes on board: red-blue - blue + red.) Start on starting point and start with red, subtract blue, subtract blue again, then end by adding red.

S. a. I don't understand.
b. I don't either.

T. (Explains again slower.) Let's do a stick line on the board using a pencil and subtracting a marker from the "pencil".

S. Abby then marks at board - marker



T. Can any one do this another way?

S. Marks space at dotted line.

T. If I measured, they'd be the same. John's is more helpful because piece left over is from the starting point. Who can tell me what John did?

S. He put the marker where pencil ended.

T. Yes. He began next step where last step ended. Now add r from where? (refers to board r-u-u+r)

S. The starting point.

T. Yes and subtract blue from where?

S. Where r ended.

T. Subtract blue from where last blue ended. What next?

S. Add the red.

T. How many think they'll be on right or left when finished? O.K. Go!

S. Pupils work on individual papers with colored strips at desks.

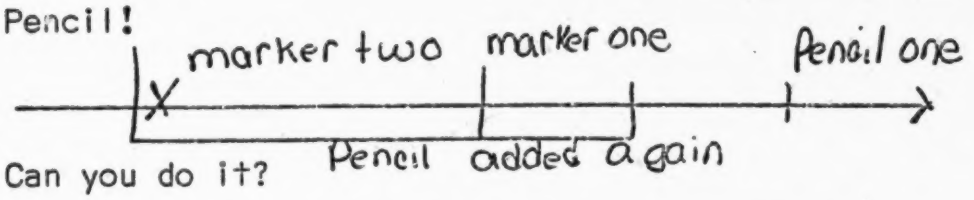
T. You can whisper the answer to me if you think you know it.

T. We'll have everyone check now. Let's see how it works on board. Let's say I start with a pencil. Now we'll subtract a marker and subtract a marker again, then add a pencil. Who wants to add pencil first?

S. a. Eric adds pencil to line at board.
b. Sandy subtracts marker
c. Kathy subtracts 2nd marker (Another child checked each of 4 steps)

T. Now we'll add what?

S. Pencil!



T. Can you do it?

S. Adds pencil from starting point.

T. Is that right?

S. a. No. She didn't start where marker ended.
b. Does this correctly.

T. Did we end at left or right of starting point? If you used red and blue would you end on right or left of starting point?

S. Right.

T. O.K. Now do red and blue again.

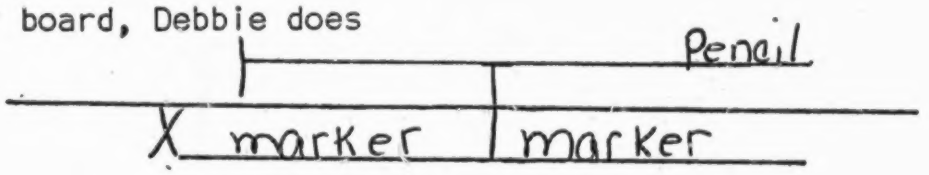
S. Mark papers at seats.

T. Here's a new one. Whisper if you know answer. Start with blue and blue, subtract red.

S. Pupils work at 2nd line on worksheet.

T. Label pencil as red (for the board) and label marker as blue.

S. At board, Debbie does



T. Who can check Debbie?

S. Noreen checks. It's o.k.

T. Who can do next step?

S. a. James does next step marks marker.
b. Another child checks.

T. Now the last step.

S. a. Peter subtracts pencil.
b. Someone checks Peter's work.

T. Which side of starting point did he end up at?

S. The right?

T. Since you're so good at adding, I'd like more help. I have to make a ruler marked off in equal units. What can I do?

S. You can get something to measure with.

T. O.K. We'll use blue as our unit of measure. What will you do?

S. Mark it with a pencil.

T. Can someone do it on the board?

S. Marks it at board.

T. What will we call the first mark?

S. Zero.

T. You come and mark it.

S. Erases X and marks starting point "0" -

T. What will we call our first unit?

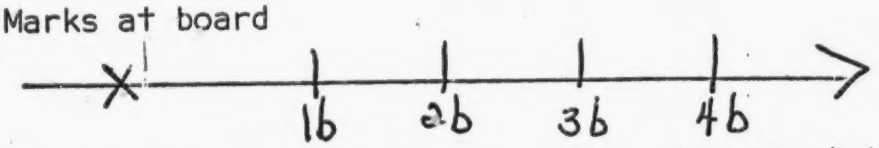
S. One.

T. To remember our unit, we can call it one what?

S. One blue.

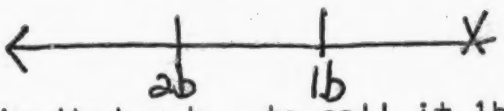
T. Write one blue. Who can mark and show me 4 b.?
Everyone can do it at desks.

S. Marks at board



T. Very good. Who can mark on other side of starting point? We'll mark off 2 steps in the other direction.

S. Marks at board



T. Is that o.k. to call it 1b and 2b?

S. You could make some red.

T. But is it a red or a blue length?

S. Blue.

T. How could I tell which side I end up at?

S. Mark it 5b.

T. How many times did we add the marker to itself?

S. Five.

T. But we have to get it straight. What can we call the marks to the left?

S. a. Negative.
b. Negative 1 b and negative 2 b.

T. If I keep marking what will I have here. (Shows imaginary point)

S. Negative 10

T. And here?

S. Positive zero.

T. I have one more problem. Sometimes I have to break down my unit to measure smaller nails. Sometimes when I add 2 nails it's only 1 unit. How can I measure?

S. Put lines in between.

T. How would we know where to put the lines?

S. Use a ruler.

T. But I'm making a ruler. I don't have one. Can you tear up paper so that added to itself it will go from starting point to 1 b? Add it to itself. Add 1 piece to itself.
Who can do it at board with this white paper? Who did it at seats? (Children work at seats.)

S. Tries at board moving oak tag strip.

T. Good. How did you do it?

S. a. I kept folding till I got the right size.
b. I have another way. Folds paper for measurement (a little too long).

T. Who can make it exactly right?

S. Linda does by holding X to 1 b and tears off scrap at end. She keeps tearing until exact. Tears off scrap at end + puts it on 1 b then folds so ends match. Marks one piece and then another piece to come to 1 b.

T. Here's what she did. She took the white paper and tore it at 1b. She folded it till the ends matched. Then she tried it and it fit. What shall we call this new mark?

S. a. Positive zero.
b. Positive 1

T. No. One is here.

S. Positive 1/2.

T. You can use this strip of paper to find $\frac{2}{2}$, $\frac{3}{2}$, $\frac{-1}{2}$, and many other places by adding the strip to itself.